# Chapter 2. Linear Equations in One Variable

## Question 1

**Solve the equation and check your solution:** $\frac{3x+5}{2x+7} = 5$

## Solution:

We have: $\frac{3x+5}{2x+7} = 5$

Multiplying both sides of the equation by $(2x + 7)$, we have

$$\frac{3x+5}{(2x+7)} \times (2x+7) = 5 \times (2x+7)$$

$$3x + 5 = 10x + 35$$

$$3x - 10x = 35 - 5 \quad \text{(Transposing the terms 10x and 5)}$$

$$-7x = 30$$

$$\frac{-7x}{-7} = \frac{30}{-7} \quad \text{(Dividing both sides by } -7\text{)}$$

$$x = -\frac{30}{7}$$

Thus, $x = \frac{-30}{7}$ is the solution of the given equation

Check : At $x = \frac{-30}{7}$,

$$\text{L.H.S.} = \frac{3\left(\frac{-30}{7}\right)+5}{2\left(\frac{-30}{7}\right)+7} = \frac{\frac{-90}{7}+5}{\frac{-60}{7}+7} = \frac{\frac{-90+35}{7}}{\frac{-60+49}{7}} = \frac{\frac{-55}{7}}{\frac{-11}{7}}$$

$$= \frac{-55}{7} \times \frac{7}{-11} = 5 = RHS$$

Hence, our solution is correct.

## Question 2

**Solve the equation and check your solution:** $\frac{2y+5}{y+4}=1$

**Solution:**

We have: $\frac{2y+5}{y+4}=1$

Multiplying both sides of the equation by y + 4, we have:

$$\frac{2y+5}{y+4}\times(y+4)=1\times(y+4)$$

$$2y+5 = y+4$$

$$2y - y = 4 - 5 \qquad \text{(Transposing y and 5)}$$

$$y = -1$$

Thus, y = -1 is the solution of the equation

**Check:** At y = -1,

$$\text{L.H.S.} = \frac{2y+5}{y+4}=\frac{2(-1)+5}{-1+4}=\frac{-2+5}{3}=\frac{3}{3}=1=RHS$$

Hence, our solution is correct.

## Question 3

**Solve the equation and check your solution:** $\frac{5z-3}{2z}=\frac{8}{9}$

**Solution:**

We have: $\frac{5z-3}{2z}=\frac{8}{9}$

Cross multiplying, we have:

$(5z-3)\times 9 = 8\times(2z)$

$5z\times 9-3\times 9 = 16z$

$45z-27=16z$

45z - 16z = 27 (Transposing 16z and - 27)

$29z=27$

$\frac{29z}{29}=\frac{27}{29}$ (Dividing both sides by 29)

$z=\frac{27}{29}$

Thus, $z=\frac{27}{29}$ is the solution of the given equation.

**Check:** At $z=\frac{27}{29}$,

$$\text{L.H.S.} = \frac{5z-3}{2z}=\frac{5\left(\frac{27}{29}\right)-3}{2\left(\frac{27}{29}\right)}=\frac{\frac{135}{29}-3}{\frac{54}{29}}=\frac{\frac{135-87}{29}}{\frac{54}{29}}$$

$$= \frac{\frac{48}{29}}{\frac{54}{29}}=\frac{48}{54}=\frac{8}{9}=RHS.$$

Hence, our solution is correct.

## Question 4

**Solve the equation and check your solution:** $\frac{1-9y}{19-3y}=\frac{5}{8}$

**Solution:**

We have; $\frac{1-9y}{19-3y}=\frac{5}{8}$

Cross multiplying , we have:

$8 \times (1 - 9y) = 5 \times (19 - 3y)$

$8 - 72y = 95 - 15y$

$-72y + 15y = 95 - 8$ (Transposing $-15y$ and 8)

$-57y = 87$

$\frac{-57y}{-57}=\frac{87}{-57}$ (Dividing both sides by $-57$)

$y = \frac{87}{-57}$

$y = -\frac{29}{19}$

Thus, $y = -\frac{29}{19}$ is the solution of the given equation.

**Check:** At $y = -\frac{29}{19}$

$L.H.S = \frac{1-9y}{19-3y}$

$= \frac{1-9\left(-\frac{29}{19}\right)}{19-3\left(\frac{-29}{19}\right)} = \frac{1+\frac{261}{19}}{19+\frac{87}{19}}$

$= \frac{\frac{19+261}{19}}{\frac{361+87}{19}} = \frac{\frac{280}{19}}{\frac{448}{19}} = \frac{280}{448}$

$= \frac{5}{8} =$ R.H.S

## Question 5

**Solve the equation:** $\frac{0.4z-3}{1.5z+9}=\frac{-7}{5}$

**Solution:**

$\frac{0.4z-3}{1.5z+9}=\frac{-7}{5}$

By cross multiplying , we have

$5(0.4z - 3) = -7(1.5z + 9)$

$2.0z - 15 = -10.5z - 63$

$2.0z + 10.5z = -63 + 15$ (Transposing $-10.5z$ and $-15$)

$12.5z = -48$

$\frac{12.5z}{12.5}=\frac{-48}{12.5}$ (Dividing both sides by 12.5)

$z = \frac{-48}{12.5}$ $z = \frac{-480}{125}$

$z = \frac{-96}{25}$

Thus, $z = \frac{-96}{25}$ is the solution of the given equation.

## Question 6

**Solve the equation:** $\frac{5y-3}{2y+1}=\frac{2}{5}$

**Solution:**

$\frac{5y-3}{2y+1}=\frac{2}{5}$

By cross multiplying , we have

$5(5y - 3) = 2(2y + 1)$

$25y - 15 = 4y + 2$

$25y - 4y = 2 + 15$ (Transposing $4y$ and $-15$)

$21y = 17$

$\frac{21y}{21}=\frac{17}{21}$ $y = \frac{17}{21}$

Thus, $y = \frac{17}{21}$ is the solution of the given equation.

## Question 7

**Solve the equation:** $\frac{2x}{3x+1} = -3$

**Solution:**

$\frac{2x}{3x+1} = -3$

Multiplying both sides of the above equation by (1 - 7x), we have:

$\frac{2x}{3x+1}(3x+1) = -3(3x+1)$

$2x = -9x - 3$

$2x + 9x = -3$ (Transposing $-9x$)

$11x = -3$

$\frac{11x}{11} = \frac{-3}{11}$ (Dividing both sides by 11)

$x = \frac{-3}{11}$

Thus, $x = \frac{-3}{11}$ is the solution of the given equation.

## Question 8

**Solve the equation:** $\frac{17(2-x)-5(x+12)}{1-7x} = 8$

**Solution:**

We have: $\frac{17(2-x)-5(x+12)}{1-7x} = 8$

Multiplying both sides of the above equation (1-7x), we have:

$\frac{17(2-x)-5(x+12)}{1-7x}(1-7x) = 8(1-7x)$

$17(2 - x) - 5(x + 12) = 8(1-7x)$
$34 - 17x - 5x - 60 = 8 - 56x$
$-22x - 26 = 8 - 56x$
$-22x + 56x = 8 + 26$ (Transposing 56x and 26)
$34x = 34$

$\frac{34x}{34} = \frac{34}{34}$ (Dividing by 34 both sides)

$x = \frac{34}{34}$

$x = 1$

Thus, $x = 1$ is the solution of the given equation.

## Question 9

**Solve the equation:** $\frac{y-(7-8y)}{9y-(3+4y)}=\frac{2}{3}$

**Solution:**

$$\frac{y-(7-8y)}{9y-(3+4y)}=\frac{2}{3}$$

By cross multiplying , we have

$3\{y-(7-8y)\}=2\{9y-(3+4y)\}$

$3\{y-7+8y\}=2\{9y-3-4y\}$

$3(9y-7)=2(5y-3)$

$27y-21=10y-6$

$27y-10y=-6+21$ (Transposing 10y and −21)

$17y=15$

$\frac{17}{17}y=\frac{15}{17}$ (Dividing both sides by 17)

$y=\frac{15}{17}$

Thus, $y=\frac{15}{17}$ is the solution of the given equation.

## Question 10

**The sum of two numbers is 45 and their ratio is 7:8. Find the numbers.**

**Solution:**

Let one of the numbers be x.

Then the other number will be 45 - x

By the given condition,

$$\frac{x}{45-x}=\frac{7}{8}$$

By cross multiplying , we have

$8x = 7(45 - x)$

$8x = 315 - 7x$

$8x + 7x = 315$ (Transposing - 7x)

$15x = 315$

$\frac{15x}{15}=\frac{315}{15}$ (Dividing both sides by 15)

$x = \frac{315}{15} = 21$

$x = 21$

Thus, one number is 21 and the other number is 45 - 21 = 24

**Check:** (1) Sum of the two numbers = 21 + 24 = 45

(2) Ratio of the two number = $\frac{21}{24}=\frac{7}{8}$

## Question 11

**The sum of the digits of a two digit number is 12. If the new number formed by reversing the digits is greater than the original number by 18, find the original number. Check your solution.**

**Solution:**

Let the digit in the ones place be x.

Then the digit in the tens place will be 12 – x.

Therefore, the original number = 10(12 - x) + x = 120 – 10x + x = 120 - 9x.

And, the new number = 10 x + (12 - x) = 10x + 12 – x = 9x + 12.

By the given condition,

New number = original number + 18

$9x + 12 = 120 - 9x + 18$

$9x + 12 = 138 - 9x$

$9x + 9x = 138 - 12$ (Transposing 9x and 12)

$18x = 126$

$\frac{18x}{18} = \frac{126}{18}$ (Dividing both sides by 18)

$x = 7$

Thus, ones digit is 7 and tens digit is 12 - 7 = 5. Hence, the required number is 57.

**Check:** (1) 5 + 7 = 12 is the sum of the digit.
(2) New number = 75.

Difference between the original number and new number = 75 – 57 = 18.

\ The new number is 18 more than the original number.

## Question 12

**The numerator of a rational number is less than its denominator by 3. If the numerator becomes three times and the denominator is increased by 20, the new number becomes $\frac{1}{8}$. Find the original number.**

**Solution:**

Let the denominator be x.

Then the numerator will be x -3

Thus, the given rational number = $\frac{x-3}{x}$

By the given condition, we have:

$\frac{3(x-3)}{x+20}=\frac{1}{8}$; $\frac{3x-9}{x+20}=\frac{1}{8}$

By cross multiplying , we have

$8(3x-9) = 1(x+20)$

$24x-72 = x+20$

$24x-x = 20+72$ (Transposing x and -72)

$23x = 92$

$\frac{23x}{23}=\frac{92}{23}$ (Dividing both sides by 23)

$x = \frac{92}{23}$ $x = 4$

Therefore, original number = $\frac{x-3}{x}=\frac{4-3}{4}=\frac{1}{4}$

**Check:** (1) 4-1 = 3 i.e. numerator is 3 less than denominator by 3.
(2) New numerator = $3\times 1 = 3$ and new denominator = 24
Hence, new number $\frac{3}{24}=\frac{1}{8}$

## Question 13

**Meera's mother is four times as old as Meera.After five years,her mother will be three times as old as she will be then.What are their present ages?**

**Solution:**

Let present age of Meera be x years.

Then the present age of Meera's mother = 4x years.

Meera's age after five years = (x + 5) years.

Meera's mother age after five years = (4x + 5) years.

By the given condition, we have:

$4x + 5 = 3(x + 5)$

$4x + 5 = 3x + 15$

$4x - 3x = 15 - 5$(Transposing 3x and 5)

$x = 10$

Thus, age of Meera = 10 years and that of Meera's mother = 4 x 10 = 40 years.

**Check:**

(1) Meera's age after five years = 10 + 5 = 15 years.

Meera's mother age after five years = 4 (10) + 5 = 40 + 5 = 45 years.

Therefore, Meera's mother age after five years is three times Meera's age after five years. Let present age of Meera be x years.

Then the present age of Meera's mother = 4x years.

Meera's age after five years = (x + 5) years.

Meera's mother age after five years = (4x + 5) years.

By the given condition, we have:

$4x + 5 = 3(x + 5)$

$4x + 5 = 3x + 15$

$4x - 3x = 15 - 5$(Transposing 3x and 5)

$x = 10$

Thus, age of Meera = 10 years and that of Meera's mother = 4 x 10 = 40 years.

## Question 14

**The length of a rectangle exceeds its breadth by 4 cm. If length and breadth are each increased by 3 cm, the area of the new rectangle will be 81 sq. cm more than that of the given rectangle. Find the length and breadth of the given rectangle. Check your solution.**

**Solution:**

Let the breadth of the rectangle be x cm.

Then the length of the rectangle will be (x + 4) cm.

Therefore, the area of the given rectangle = length ´ breadth

$= (x + 4)x$

$= (x^2 + 4x)\ cm^2$

By the given condition, we have:

Area of the new rectangle = Area of the given rectangle + 81 $cm^2$

\ Area of the new rectangle = New length ´ New breadth

$(x^2 + 4x) + 81 = [(x + 4) + 3](x + 3)$

$x^2 + 4x + 81 = (x + 7)(x + 3)$

$x^2 + 4x + 81 = x(x + 3) + 7(x + 3)$

$x^2 + 4x + 81 = x^2 + 3x + 7x + 21$

$x^2 + 4x + 81 = x^2 + 10x + 21$

$4x + 81 = 10x + 21$ (Cancelling $x^2$ both sides)

$4x - 10x = 21 - 81$ (Transposing 10x and 81)

$-6x = -60$

$\frac{-6x}{-6} = \frac{-60}{-6}$ (Dividing both sides by −6)

$x = 10$

Thus, the breadth of the rectangle is 10 cm and the length of the rectangle is 10 + 4 = 14 cm.

**Check:**

(1) (14-10)cm = 4cm, i.e. the length of rectangle exceeds the breadth by 4 cm.

(2) New length = 14 + 7 = 17 cm and new breadth = 10 + 3 = 13 cm.

Therefore, area of the new rectangle – area of the given rectangle

= 17 ´ 13 – 14 ´ 10

= 221 – 140

= 81 cm.

## Question 15

**An altitude of a triangle is five-thirds the length of its corresponding base. If the altitude were increased by 4 cm and the base decreased by 2 cm, the area of the triangle would remain the same. Find the base and altitude of the triangle.**

**Solution:**

Let the base of the triangle be x cm.

Then the altitude of the triangle will be $\frac{5}{3}x$ cm

Therefore, the area of the given triangle = $\frac{1}{2}$ × base × altitude

$$= \frac{5}{6}x^2$$

By the given condition, we have:

Area of the new triangle = Area of the given triangle

Therefore area of the new triangle = $\frac{1}{2}$ × new base × new altitude

$$\frac{5}{6}x^2 = \frac{1}{2}(x-2)\left(\frac{5}{3}x+4\right)$$

$$\frac{5}{6}x^2 = \frac{1}{2}\left[x\left(\frac{5}{3}x+4\right)-2\left(\frac{5}{3}x+4\right)\right]$$

$$\frac{5}{6}x^2 = \frac{1}{2}\left(\frac{5}{3}x^2+4x-\frac{10}{3}x-8\right)$$

$$\frac{5}{6}x^2 = \frac{1}{2}\left(\frac{5}{3}x^2+\frac{12x-10x}{3}-8\right)$$

$$\frac{5}{6}x^2 = \frac{1}{2}\left(\frac{5}{3}x^2+\frac{2}{3}x-8\right)$$

$$\frac{5}{6}x^2 = \frac{5}{6}x^2+\frac{1}{3}x-4$$

$$0 = \frac{1}{3}x-4 \qquad \left(Cancelling \frac{5}{6}x^2\right)$$

$0 = \frac{1}{3}x = 4$ (Transposing to the L.H.S.)

$3\times\frac{1}{3}x = 3\times4$ (Multiplying by 3 on both sides)

**Check:**

(1) Altitude of the triangle = $\frac{5}{3}\times12 = 20cm$

(2) Area of the given triangle = $\frac{1}{2}\times12\times20 = 120cm^2$

Area of the new triangle = $\frac{1}{2}\times(12-2)\times(20+4)$

$= \frac{1}{2}\times10\times24$ $= 120\ cm^2$

Thus, the base of the triangle is = 12cm

∴ Area of the new triangle is equal to the area of the given triangle.

## Question 16

**The sum of two numbers is 2490. If 6.5% of one number is equal to 8.5 % of the other, find the numbers.**

**Solution:**

Let one number be x

Then the other number will be 2490 – x.

By the given condition, we have:

$$\frac{6.5}{100} \times x = \frac{8.5}{100} \times (2490 - x)$$

$6.5x = 8.5\ (2490\text{-}x)$

$6.5x = 21165 - 8.5x$

$6.5x + 8.5x = 21165$ (Transposing 8.5x)

$15x = 21165$

$$\frac{15x}{15} = \frac{21165}{15}$$ (Dividing both sides by 15)

$$x = \frac{21165}{15}$$

$x = 1411$

Thus, one number is = 1411 and the other number = 2490 – 1411 = 1079.

**Check:** (1) 1411+1079=2490 i.e the sum of the two numbers

(2) 6.5% of $1411 = \frac{6.5}{100} \times 1411 = 0.065 \times 1411 = 91.715$

8.5% of $1079 = \frac{8.5}{\quad} \times 1079 = 0.085 \times 1079 = 91.715$

## Question 17

**Three prizes are to be distributed in a quiz contest. The value of the 2nd prize is five-sixths the value of the first prize and the value of the third prize is four-fifths that of the 2nd prize. If the total value of the three prizes is Rs.150, find the value of each prize.**

**Solution:**

Let the value of first prize be Rs. x.

Then, the value of the second prize will be Rs. $\frac{5}{6}$ x

and the value of the third prize will be Rs $\frac{4}{5}\left(\frac{5}{6}x\right)$ = Rs. $\frac{2}{3}x$

By the given condition, we have:

$$x + \frac{5}{6}x + \frac{2}{3}x = 150$$

$$6 \times \left(x + \frac{5}{6}x + \frac{2}{3}x\right) = 6 \times 150 \quad \text{(Multiplying both sides by 6)}$$

$$6x + 5x + 4x = 900$$

$$15x = 900$$

$$\frac{15x}{15} = \frac{900}{15} \quad \text{(Dividing by 15 on both sides)}$$

$$x = 60$$

Thus, the value of first prize = Rs.60,

second prize = Rs. $\frac{5}{6}(60)$ = Rs.5 × 10 = Rs.50.

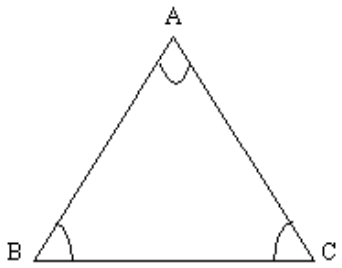
and third prize = Rs. $\frac{2}{3}(60)$ = Rs.40

**Check:** (1) Rs. (60 + 50 + 40) = Rs.150 i.e. the total value of the three prizes.

## Question 18

**One of the angles of a triangle is equal to the sum of the other two angles. If the ratio of the other two angles is 4:5, find the angles of the triangle.**

**Solution:**




Let one of the angle be ĐA and the other two angles are ∠B and ∠C

Given one of the angles of a triangle is equal to the sum of the other two angles

i.e., ∠A = ∠B + ∠C

Also it is given the ratio of the other two angles is 4:5

Therefore ,the other two angles B and C be 4x and 5x

By the given condition, we have

$$\angle A = \angle B + \angle C$$

$$\therefore \angle A = 4x + 5x = 9x$$

We know that, the sum of the angles of a triangle is 180°.

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$\therefore 9x + 4x + 5x = 180^\circ$$

$$18x = 180$$

$$x = \frac{\cancel{180}^{10}}{\cancel{18}_{1}}$$

$$x = 10$$

Thus, the angles of the triangle are 9´10, 4´10, 5´10 i.e. 90°, 40°, 50°.

**Check:**

(1) 40° + 50° = 90° i.e. one of the angles of a triangle is equal to the sum of the other two angles.

(2) 40:50 = 4:5 is the ratio of the other two angles.

## Question 19

**Solve for x.**

(i) $3x - 5 = x + 5$

(ii) $9x - 3 = 7x + 3$

(iii) $\frac{(3x+2)}{(2x-3)} = 8$

(iv) $\frac{(3x-8)}{(4x+4)} = \frac{9}{28}$

**Solution:**

(i) $3x - 5 = x + 5$

$3x - x = 5 + 5$

$2x = 10$

$x = 5$

(ii) $9x - 3 = 7x + 3$

$9x - 7x = 3 + 3$

$2x = 6$

$x = 3$

(iii) $\frac{(3x+2)}{(2x-3)} = 8$

$3x + 2 = 8(2x - 3)$

$3x + 2 = 16x - 24$

$3x - 16x = -24 - 2$

$-13x = -26$

$x = 2$

(iv) $\frac{(3x-8)}{(4x+4)} = \frac{9}{28}$

$28(3x - 8) = 9(4x + 4)$

$84x - 224 = 36x + 36$

$84x - 36x = 36 + 224$

$48x = 260$

$x = 65/12$

## Question 20

**In a students' hostel, one third of the total number of girls and four more take vegetarian food only, one fourth and three more take non-vegetarian food only. The other 103 take both vegetarian and non-vegetarian food. How many girls are there in the hostel?**

**Solution:**

Let the total number of girls be x.

No. of girls who take vegetarian food only = $\frac{1}{3}x + 4$

No. of girls who take non - vegetarian food only = $\frac{1}{4}x + 3$

No. of girls who take both vegetarian and non - vegetarian food = 103.

By the given condition,

$$\frac{1}{3}x + 4 + \frac{1}{4}x + 3 + 103 = x$$

$$\frac{1}{3}x + \frac{1}{4}x - x + 110 = 0$$

Multiply through out by 12,

$$4x + 3x - 12x + 1320 = 0$$

$$-5x + 1320 = 0$$

$$-5x = -1320$$

$$x = \frac{1320}{5}$$

$x = 264 \therefore$ There are 264 girls in the hostel.

## Question 21

**The sum of the digits of a two digit number is 9. If the digits are reversed,the number is 63 more than the original. Find the number.**

**Solution:**

Let the units digit be x.

Then the tens digit is 9 - x.

Therefore the original number is 10(9 - x) + x = 90 - 10x + x = 90 - 9x.

On reversing the order of the digits the number obtained is 10x + 9 - x = 9x + 9.

By the given condition,

$9x + 9 = 63 + (90 - 9x)$

$(9x + 9) - (90 - 9x) = 63$

$9x + 9 - 90 + 9x = 63$

$18x = 144$ (Transposing 9 and -90 to the R.H.S.)

$x = 8$

Therefore the original number is $90 - 9\times 8 = 90 - 72 = 18$.

## Question 22

**A mother is 46years old and her son is 21 years old in the year 1997. In what year was the mother six times as old as the son?**

**Solution:**

Age of the mother in the year 1997 = 46years

Age of her son in the year 1997 = 21years

x years ago, age of the mother = 46 - x.

x years ago, age of the son = 21 - x.

Hence by the given condition,

$46 - x = 6(21 - x)$

$46 - x = 126 - 6x$

$-x + 6x = 126 - 46$

$5x = 80$

$x = 16$

Thus the year was 1997-16 = 1981.

## Question 23

**A man sold a bicycle for an amount, which was greater than Rs.988 by half the price he paid for it, and made a profit of Rs.300. How much did he buy the bicycle for?**

**Solution:**

Let the CP be Rs.x

Then SP = $988 + \frac{1}{2}x$

Profit = Rs. 300

By the given condition,

SP = CP + Profit

$988 + \frac{1}{2}x = x + 300$

Multiplying by 2 on both sides

$2 \times 988 + x = 2x + 600$

$1976 + x = 2x + 600$

$1976 - 600 = 2x - x$

$\therefore x = 1376$

## Question 24

**The unequal side of an isosceles triangle is 3cm more than one of its equal sides. If the perimeter of the triangle is 18cm, find the length of the three sides.**

**Solution:**

Let the length of the equal sides be x cm.

Then the length of the unequal sides will be (x + 3) cm.

Perimeter of an isosceles triangle is 18.$\therefore x + x + 3 + x = 18$

$3x + 3 = 18$

$3x = 15 \qquad \therefore x = 5$

Therefore the equal sides of the triangle is 5cm and unequal side is 8cm.

## Question 25

**The sum of 5 consecutive numbers is 140. Find the numbers.**

**Solution:**

Let the 5 consecutive numbers be x, x+1, x+2, x +3, x + 4.

$x + x + 1 + x + 2 + x + 3 + x + 4 = 140$

$5x + 10 = 140$

$5x = 130$

$x = 26$

Therefore the numbers are 26, 27, 28, 29, 30.

## Question 26

**The tens and units digits of a number are the same. When the number is added to its reverse, the sum is 110. What is the number?**

**Solution:**

Let the units digit be x.

Then the tens digit is also x.

Therefore the number is $10x + x = 11x$.

On reversing the order of the digit the number is $10x + x = 11x$.

Hence by the given condition we have,

$11x + 11x = 110$

$22x = 110$

$x = 5$

Therefore the required number is 55.